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Optimum Design of Structures at Elevated Temperatures

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This paper describes optimum design techniques for structures where high temperatures are important design considerations. An improved variant of fully stressed design (FSD), denoted thermal fully stressed design (TFSD), is described which converges significantly faster than FSD for problems where thermal stresses are comparable in magnitude to mechanical stresses. Also, an optimality criterion is described for sizing structures subjected to temperature constraints. Simultaneous requirements on strength and temperatures are handled by two different techniques: 1) state-of-the-art mathematical programming method and 2) the optimality criterion technique for temperature constraints combined with FSD for strength constraints. Finally, a method is described to optimize the insulation and ply thickness of an insulated composite panel with a time-varying temperature applied to the outer surface of the insulation and a general set of in-plane loads applied to the panel. Temperature and strength constraints are satisfied at a specified number of times. Only a small number of such discrete times are necessary for adequate results for an example contained herein.

Nomenclature

A	= area of a bar element
a	= design variable
b	$= 2\sigma_{xT}\sigma_{xM} + 2\sigma_{yT}\sigma_{yM} - \sigma_{xT}\sigma_{yM} - \sigma_{yT}\sigma_{xM} + 6\sigma_{xyT}\sigma_{xyM}$, thermal-mechanical coupling term
C	= conductivity matrix
c	= heat capacity
E	= Young's modulus
F_1	$= 1/X_T + 1/X_C$
F_2	$= 1/Y_T + 1/Y_C$
F_{11}	$= -1/X_T X_C$
F_{22}	$= -1/Y_T Y_C$
F_{66}	$= 1/S^2$
g	= design constraint
G_{12}	= shear modulus
ℓ	= bar length
N_x, N_y, N_{xy}	= forces per unit length
n	= iteration number
Nb	= number of bar elements
NT	= number of temperature constraints
N	= number of design variables
NC	= total number of constraints
P	= pressure
$\dot{Q}$	= heat flux
q	= thermal load vector
S	= shear strength
s	= elemental temperature vector for dummy thermal load
T	= temperature
T_{eq}	= temperature applied to outer surface of panel insulation
T^*	= time-integrated average temperature
t	= membrane thickness
t_I	= insulation thickness
t_0, t_{45}, t_{90}	= thickness of 0-, ±45-, and 90-deg plies, respectively
V	$= [\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\sigma_{xy}^2]^{1/2}$, von Mises stress
w	= mass
X_T, X_C	= tensile and compressive strengths in fiber direction
Y_T, Y_C	= tensile and compressive strengths in transverse direction
α	= coefficient of thermal expansion
λ	= Lagrange multiplier
ρ	= mass density
ρ_I	= density of insulation material
ρ_2	= density of structural material
ν	= Poisson's ratio
τ	= time
$\bar{\tau}$	= period of time that constraints are enforced
$\sigma_x, \sigma_y, \sigma_{xy}$	= stress components
σ	= uniaxial stress

Subscripts

a	= allowable
i	= iteration number, discrete time number
j	= layer number
k	= temperature constraint number
M	= mechanical
m	= element number
n	= interation number
T	= thermal
$1,2$	= parallel and transverse to fibers, respectively

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Introduction

THE necessity for lightweight aerospace structures to withstand both mechanical loading and severe thermal environments in applications such as atmospheric entry and high-speed cruise has led to the problem of designing minimum-mass structures to satisfy both structural and thermal design requirements. Previous work in structural synthesis for minimum mass has concentrated on sizing mechanically loaded structures, and a great deal of methodology has been developed for that class of problems. Applications of fully stressed design (FSD), optimality criteria methods, and mathematical programming techniques abound in the technical literature (see for example Refs. 1-11). Generally, the aforementioned methods have not been extended to problems where thermal loads and thermal design requirements are significant considerations. Therefore, an effort was undertaken to apply and adapt some of the methods of optimum structural design for such a class of problems. The purpose of this paper is to summarize recent progress and describe some applications of the methods.

Initial work dealt with separate treatment of strength and temperature constraints. A variant of FSD denoted TFSD (thermal fully stressed design) was developed for strength constraints and found to be advantageous where thermal stresses are comparable in magnitude to mechanical stresses.[12,13] For problems where only temperature constraints are enforced, a method of minimum-mass design was developed using an optimality criterion approach.[14]

Efforts were then focused on the coupled problem of optimizing a heated and loaded structure subject to strength and temperature constraints using mathematical programming and optimality criteria techniques. In Ref. 15, coupled thermal-structural optimization for structures modeled with finite elements was described using state-of-the-art mathematical programming techniques. New applications of the techniques of Ref. 15 are included in the present paper. A recently developed alternative procedure for coupled problems based on concepts set forth in Refs. 16 and 17 combines the optimality criterion method for temperature constraints with FSD for strength constraints. Results from this approach also are presented.

A topic of current emphasis is development of methods for thermal/structural design problems under transient loads, which requires that constraints be satisfied over a period of time. Work has been initiated to optimize an insulated structural panel subject to mechanical forces and a time-dependent temperature at the outer surface of the insulation. A method of handling the time-varying constraints has been developed in which the constraints are satisfied at a specified number of times.

Thermal Fully Stressed Design (TFSD)

Because of its computational efficiency and convenience, fully stressed design (FSD) is widely used to size structures for strength constraints. When applied to structures under mechanical loads plus prescribed temperatures FSD demonstrates slow convergence when thermal stresses are comparable in magnitude to mechanical stresses. The slow convergence of FSD for structures with large thermal stresses is associated with the relative insensitivity of thermal stresses to structural sizing.[12] In this section, the algorithms for sizing bars and isotropic membrane elements are described and their application to a wing box is presented.

Algorithm for Uniaxial (Bar) Elements

The theoretical basis of the TFSD algorithm is only slightly different from that of FSD. The difference is in how the thermal stress changes as a function of member size from iteration to iteration. In the FSD method, the basis for structural resizing is the total stresses and is predicated on maintaining a constant force in each member during an iteration. The basis for the thermal fully stressed design (TFSD) algorithm is that during each iteration the thermal stress and the mechanical force each remain constant.

For a change in member size from the ith to the $(i+1)$th iteration the algorithm is

$$A_{i+1} = [\sigma_{Mi}/(\sigma_a - \sigma_{Ti})]A_i \qquad (1)$$

In Eq. (1), σ_M is the stress due to mechanical loads acting alone, σ_T is the stress due to thermal loads acting alone, and σ_a is either the tensile or compressive allowable stress, depending on the sign of σ_M. For later reference, the usual FSD algorithm is given by

$$A_{i+1} = [\sigma_{Mi} + \sigma_{Ti})/\sigma_a]A_i \qquad (2)$$

where σ_a is either the tensile or compressive allowable stress, depending on the sign of the total stress as given by the numerator.

Algorithm for Isotropic Membrane Elements

The Von Mises failure criterion for isotropic membrane elements is

$$V(\sigma_x,\sigma_y,\sigma_{xy}) = [\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\sigma_{xy}^2]^{1/2} = \sigma_a \qquad (3)$$

where σ_x and σ_y are direct stresses along orthogonal coordinate directions in the plane of the element; σ_{xy} is the shear stress on that plane; and σ_a is the allowable stress. The sizing formula is derived by generalizing Eq. (1) which may be rearranged as follows:

$$\frac{\sigma_{Mi}}{A_{i+1}/A_i} + \sigma_{Ti} = \sigma_a \qquad (4)$$

By analogy, the corresponding statement for biaxially stressed members is

$$V\left(\frac{\sigma_{xMi}}{r_i} + \sigma_{xTi}, \frac{\sigma_{yMi}}{r_i} + \sigma_{yTi}, \frac{\sigma_{xyMi}}{r_i} + \sigma_{xyTi}\right) = \sigma_a \qquad (5)$$

where $r_i = t_{i+1}/t_i$ and t is the element thickness. Expanding Eq. (5) and solving the resulting equation by the quadratic formula gives the resizing algorithm

$$t_{i+1} = \left[\frac{b_i}{2(\sigma_a^2 - V_{Ti}^2)} + \sqrt{\frac{b_i^2}{4(\sigma_a^2 - V_{Ti}^2)^2} + \frac{V_{Mi}^2}{\sigma_a^2 - V_{Ti}^2}}\right]t_i \qquad (6)$$

Repeated use of Eq. (6) drives each member toward the fully stressed condition

$$V_M^2 + V_T^2 + b = V^2 = \sigma_a^2 \qquad (7)$$

As an alternative to Eq. (6) one might be tempted to generalize Eq. (1) to the biaxial stress case by using the formula

$$t_{i+1} = [V_{Mi}/(\sigma_a - V_{Ti})]t_i \qquad (8)$$

However, it is clear that use of Eq. (8) will drive members toward the condition

$$V_M + V_T = \sigma_\alpha \qquad (9)$$

which is obviously not the condition of a fully stressed member [Eq. (7)].

For later reference, the corresponding FSD resizing algorithm used to compare results with TFSD is

$$t_{i+1} = (V_i/\sigma_a)t_i \qquad (10)$$

Application to a Wing Box

To illustrate the application of TFSD and to compare the algorithm with ordinary FSD, calculations were carried out using a computer program incorporating both the TFSD and FSD algorithms. Finite-element methods using standard bar elements and the "TRIM 6" (Ref. 18) triangular membrane elements were used for the analysis. The configuration considered is the aluminum wing box depicted in Fig. 1. The wing box is built-in at the root. The loads consist of in-plane forces, normal pressure applied to the upper and lower surfaces, and a temperature gradient through the depth of the structure. The loads are summarized in Table 1, and the material properties are given in Table 2. The finite-element model of the structure consists of 68 bars, 8 membranes, and 30 grid points and is described in Ref. 15. Elements on the lower surface are assigned material properties corresponding to a temperature of 177°C (350°F). Elements on the upper surface are assigned room temperature properties, i.e., the small degradation in material properties at 85°C (185°F) is neglected. Elements joining the upper and lower surfaces have properties that are averages of the upper and lower values.

Identical final designs (with a mass of 105 kg (231 lbm)) were obtained by TFSD and FSD. Both algorithms started with unit values for all design variables. The TFSD algorithm required only two iterations to converge within 5% of the final mass; FSD required five iterations for the same degree of convergence. Figure 2 shows the convergence of TFSD and FSD with separate curves for the mass of the bars and membranes and the total mass. The principal difference in the performance of TFSD and FSD is the convergence of the mass of the membrane elements; and this is due to the high thermal stresses in the membranes as compared with the bars. It is, of course, a characteristic of TFSD that its performance relative to FSD improves with increasing thermal stress levels.

Additional experience with TFSD and numerous comparisons with FSD have been documented elsewhere (see Refs. 12 and 13). It is concluded from such studies that the TFSD algorithm is worthy of consideration as an alternative to FSD for structures with significant thermal stresses.

Optimality Criterion for Temperature Constraints

The need for a method to size structures based solely on temperature constraints led to consideration of optimality criteria. Optimality criterion methods have been developed for sizing structures for a variety of constraints including strength, displacement, flutter, and buckling.[4-6] Such methods share the convenience with FSD of having explicit

Table 1 Mechanical and thermal loads on wing box

	Upper surface	Lower surface
N_x		
kN/m	−58.8	−700
lbf/in.	−336	−4000
N_y		
kN/m	−210	228
lbf/in.	−1200	1300
N_{xy}		
kN/m	92	22.4
lbf/in.	528	128
P		
Pa	276	6895
psi	0.04	1.0
ΔT[a]		
°C	43	135
°F	110	275

[a] $\Delta T = T - T_0$; $T_0 = 24°C$ (75°F).

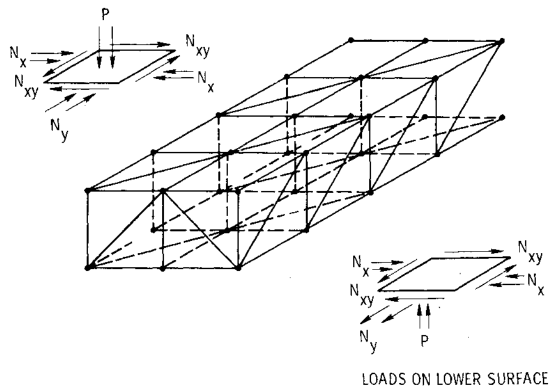


Fig. 1 Finite-element model of wing box with applied loading.

Table 2 Material properties used in calculations

Property	Graphite/polyimide RT	Graphite/polyimide 260°C (500°F)	Graphite/epoxy RT	Graphite/epoxy 177°C (350°F)	Aluminum RT	Aluminum 177°C (350°F)
E_1						
GPa	133	133	155	155	73.0	69.6
psi	19.3×10^6	19.3×10^6	22.5×10^6	22.5×10^6	10.6×10^6	10.1×10^6
E_2						
GPa	9.10	4.14	8.82	5.58		
psi	1.32×10^6	0.6×10^6	1.28×10^6	0.81×10^6	...	...
ν_{12}	0.37	0.51	0.30	0.23	0.33	0.33
G_{12}						
GPa	5.58	4.41	5.10	0.551		
psi	0.81×10^6	0.64×10^6	0.74×10^6	0.08×10^6	...	...
α_1						
°C^{-1}	-0.68×10^{-6}	0.144×10^{-6}	-0.234×10^{-6}	-0.126×10^{-6}	22.77×10^{-6}	23.8×10^{-6}
°F^{-1}	-0.38×10^{-6}	0.08×10^{-6}	-0.13×10^{-6}	-0.07×10^{-6}	12.65×10^{-6}	13.8×10^{-6}
α_2						
°C^{-1}	27.0×10^{-6}	45.0×10^{-6}	34.0×10^{-6}	78.7×10^{-6}		
°F^{-1}	15.0×10^{-6}	25.0×10^{-6}	18.9×10^{-6}	43.7×10^{-6}	...	...
X_t						
GPa	1.08	1.02	1.11	1.03	0.400	0.320
psi	157,400	147,300	161,000	150,000	58,000	46,400
X_c						
MPa	−867	−450	−972	−848	−0.400	−0.300
psi	−125,800	−65,200	−141,000	−123,000	−58,000	−43,500
Y_t						
MPa	16.5	6.62	35.8	30.0		
psi	2390	960	5190	3040	...	...
Y_c						
MPa	−109	−87.8	−170	−119		
psi	−15,790	−12,730	−24,700	−17,300	...	...
S						
MPa	93.8	53.1	57.9	25.9		
psi	13,600	7700	8400	3760	...	...
ρ						
kg/m^3	1550		1550		2800	
lbm/in.^{3}	0.056		0.056		0.101	

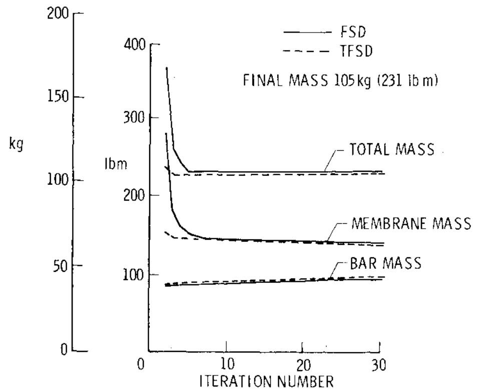


Fig. 2 Convergence of TFSD and FSD for aluminum wing box. (Initial design not plotted.)

resizing formulas and are useful when a single type of constraint is involved. Thus the problem of mass minimization with temperature constraints is well suited to solution by an optimality criterion approach. For the case where temperatures are controlled by altering conduction paths, the task was eased considerably by an existing algorithm based on an optimality criterion for displacement constraints[5] and the analogy between the matrix equations governing temperatures and displacements. It was merely necessary to adapt the optimality criterion for displacement constraints to temperature constraints. Such a development is described in Ref. 14 and summarized in this section.

Using Lagrange multipliers, the problem is posed formally as follows: minimize

$$w^* = w + \sum_{k=1}^{NT} \lambda_k (T_k - T_{a,k}) \tag{11}$$

where w is the mass of the structure, T_k is the kth constrainted temperature, and $T_{a,k}$ is the allowable temperature at the kth point. The necessary condition for an optimum design is obtained by equating to zero the first partial derivatives of w^* with respect to the design variables. Consider a structure represented by simple bar thermal finite elements. The structural mass may be written

$$w = \sum_{m=1}^{Nb} \rho_m A_m \ell_m \tag{12}$$

where ρ is the bar mass density, A is the bar area (the design variable) and ℓ is the bar length. Structural temperatures are calculated from the following equation

$$[C]\{T\} = \{q\} \tag{13}$$

where $[C]$ is a conductivity matrix, $\{T\}$ is a vector of unknown temperatures, and $\{q\}$ is an applied thermal load vector. Application of the necessary condition for an optimum design to Eq. (11) leads to

$$\rho_m \ell_m + \sum_{k=1}^{NT} \lambda_k \frac{\partial T_k}{\partial A_m} = 0 \tag{14}$$

Differentiation of Eq. (13) with respect to the design variables leads to the following expression for $\partial T_k / \partial A_m$:

$$\frac{\partial T_k}{\partial A_m} = -\{T\}_m^T \frac{[C]_m}{A_m} \{s\}_{m,k} \tag{15}$$

where $\{T\}_m$ is the elemental temperature vector for the mth element, $[C]_m$ is the elemental conductivity matrix, and $\{s\}_{m,k}$ is an elemental temperature vector obtained by solution of Eq. (13) for a unit thermal load applied at the kth constraint point. Equation (14) may be rewritten as

$$\sum_{k=1}^{NT} \lambda_k \{T\}_m^T [C]_m \{s\}_{m,k} / \rho_m \ell_m A_m = 1 \quad m = 1,2,\ldots,Nb \tag{16}$$

Equation (16) is the necessary condition for an optimum design. The following simple iteration scheme is used in the optimization process to satisfy Eq. (16)

$$A_{m,n+1} = (E_m / \rho_m \ell_m)^{\gamma_1} A_{m,n} \tag{17}$$

where

$$E_m = \sum_{k=1}^{NT} \frac{\lambda_k \{T\}_m^T [C]_m \{s\}_{m,k}}{A_m} \tag{18}$$

The Lagrange multipliers are also obtained by iteration as follows

$$\lambda_{k,n+1} = (T_k / T_{a,k})^{\gamma_2} \lambda_{k,n} \tag{19}$$

In the iteration process, the exponent γ_1 controls the design variable step size and γ_2 controls the convergence rate of Eq. (19). Similar expressions for two-dimensional thermal finite elements are given in Ref. 14. As formulated, the optimality criterion and iteration formulas apply strictly to equality constraints; however, the more practical case of inequality constraints is also accommodated by the same formulas, since for passive constraints, the corresponding values of λ_k approach zero and their impact on the resizing equations becomes negligible.

To assess the accuracy of the thermal optimality criterion technique and especially to ascertain the effects of convective heat transfer and multiple temperature constraints on the accuracy and convergence characteristics of the resizing formulas, the method has been used to size square and triangular plates.[14] As described in Ref. 14, the final designs were verified by designs using the general purpose optimizer CONMIN.[19,20] Essentially identical designs were obtained by

Table 3 Optimized volume of heated plates

Problem	Dimensionless volume (optimality criterion)	Dimensionless volume (math programming)	No. iterations for optimality criterion
Square plate conduction only	0.0474	0.0479	22
Square plate conduction and convection	0.0343	0.0342	50
Triangular plate, single constraint	0.0249	0.0252	25
Triangular plate, multiple constraints	0.0316	0.0312	45

both methods. Table 3 contains a summary of these results and indicates that the two methods agree within 1.5% for the optimum mass. Additionally, neither convective heat-transfer effects nor multiple temperature constraints affect the accuracy of the final designs, but both have a tendency to slow down convergence, as indicated in Table 3.

The thermal optimality criterion and sizing formula provides a fast and accurate procedure for sizing structures based solely on thermal considerations. Such a capability had not previously existed. Further, as shown in the next section, this procedure when combined with TFSD or FSD for stress constraints, forms a powerful tool for coupled thermal structural optimization.

Coupled Thermal-Structure Optimization

In the two previous sections of this paper, methods were discussed for sizing structures for strength and temperature requirements separately. The next logical need is for structural sizing methods by which stresses and temperatures are controlled simultaneously, i.e., coupled thermal-structural optimization.

Mathematical Programming Method

The generality and state of development of nonlinear mathematical programming methods permit a logical approach to the coupled problem. Such items as search procedures and representation of constraints are available in general purpose optimizers.[20,21] For applications to coupled thermal-structural optimization the main work is to choose an optimizer, define the objective function and constraints, and join an appropriate analysis code to the optimizer.

The present optimization problem consists of determining the set of bar areas and membrane thicknesses which minimizes the mass of a structure w subject to a set of constraint functions $\{g\}$. The constraint functions define requirements that temperatures and stresses remain below prescribed limits. The problem is solved by the sequence of unconstrained minimizations technique (SUMT) in which constraints are incorporated by an exterior penalty function.[21] The formal statement of the problem is: Minimize

$$\phi = w + \sum_{j=1}^{NC} R_j \langle g_j \rangle^2 \tag{20}$$

for a sequence of increasing values of the weighting factors R_j. The constraint functions are defined as follows:

Bar Stress

$$g = \sigma / \sigma_a - 1 \tag{21}$$

Membrane Stress

$$g = V / \sigma_a - 1 \tag{22}$$

Grid Point Temperature

$$g = T / T_a - 1 \tag{23}$$

The notation $\langle g_j \rangle$ is a penalty term defined as follows

$$\langle g_j \rangle = \begin{cases} g_j & \text{if } g_j \geq 0 \\ 0 & \text{if } g_j < 0 \end{cases} \tag{24}$$

Positive values of g_j represent violated constraints and result in nonzero values of $\langle g_j \rangle$. Negative and zero values of g_j represent satisfied constraints and give zero values for $\langle g_j \rangle$.

Stresses and temperatures needed to evaluate the constraints are calculated using a finite-element analysis model containing bar and TRIM-6 triangular membrane elements.[18]

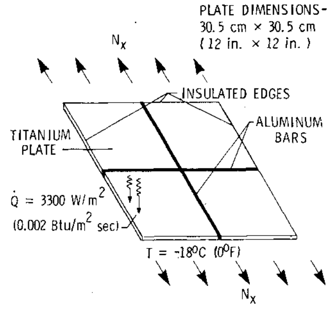
The minimizations of the function in Eq. (13) are carried out by use of a general-purpose optimizer denoted AESOP.[21] The AESOP program contains several optimization algorithms that can be selected in various combinations by the user. The approach used here was a combination called "creeper" and "pattern." These algorithms are described in Ref. 21.

Optimality Criterion Combined with FSD

Although math programming procedures are applicable to a general class of structural optimization problems, explicit resizing algorithms such as FSD, TFSD, or optimality criteria methods are usually preferred when a single type of constraint is involved. When more than one type of constraint is involved, it is usually necessary to resort to mathematical programming. Recently, it was demonstrated[16,17] that simultaneous treatment of strength constraints by FSD and either displacement or buckling constraints by an optimality criterion is feasible. Such results led to an alternate coupled thermal-structural optimization procedure by combining FSD (or optionally TFSD) with the previously described optimality criterion for temperature constraints. At this writing only the FSD option has been implemented. The technique of the method is straightforward, the bar elements are resized by either Eqs. (17) or (2) (whichever gives the larger area). Analogous logic is used for the two-dimensional finite elements. There is no assurance that the design obtained by such a procedure always gives the minimum-mass design. However, engineering intuition and results of Refs. 16 and 17 suggest that reasonably good designs are achievable. Direct validation of this approach is provided by the results described in the next section.

Application of Coupled Thermal Structural Optimization Procedures

Both the math programming and the alternate procedure are applied to the configuration shown in Fig. 3, which consists of a titanium panel with aluminum bars. This configuration is representative of a class of structures where one material satisfies strength requirements and the other acts as an efficient conductor to transfer incident heat to a heat sink. A particular example of such an application is an actively cooled panel of a hypersonic cruise aircraft.[22,23] Although the present model is not intended to be an accurate model of an actively cooled panel, the intent is to depict the essential features of this design concept. The heat sink is represented by the edge maintained at $T = -18°C$ (0°F). Loads applied to the structure include a force per unit width N_x shown in Fig. 3. Two different values of N_x were used in the calculations, 210 kN/m (1200 lbf/in.) and 6.30 MN/m (36,000 lbf/in.). The heat load consists of a uniform heat flux of 3000 W/m^2 (0.002 Btu/in.^{2} s) over the surface of the plate. Material properties are given in Fig. 3b. The allowable temperatures are 177°C (350°F) for aluminum and 260°C (500°F) for titanium. The finite-element model is shown in Fig. 4 and



a) CONFIGURATION AND LOADS

PROPERTY		ALUMINUM	TITANIUM
E	GPa	73.0	103
	psi	10.1×10^6	15×10^6
α	°C^{-1}	28.3×10^{-6}	10.4×10^{-6}
	°F^{-1}	13.2×10^{-6}	5.8×10^{-6}
ρ	kg/m^3	2770	4437
	lbm/in.^{3}	0.100	0.16
k	$\frac{W}{m\ °C}$	234	9.49
	$\frac{Btu}{in.\text{-}sec\text{-}°F}$	0.00313	0.000127
σ_a	MPa	320	700
	psi	46400	101500
T_a	°C	177	260
	°F	350	500

b) MATERIAL PROPERTIES

Fig. 3 Titanium plate with aluminum bars used to demonstrate coupled optimization procedures.

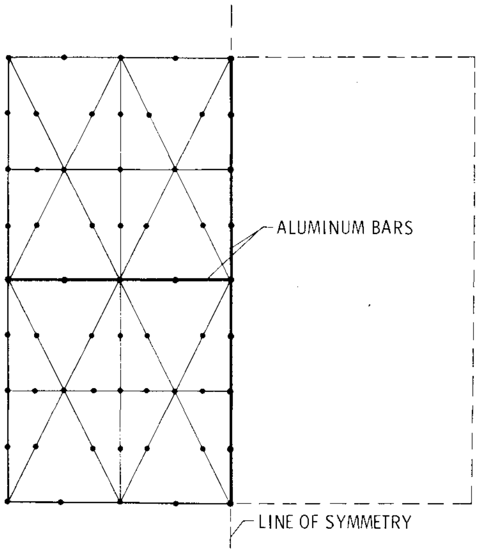


Fig. 4 Finite-element model of titanium plate with aluminum bars.

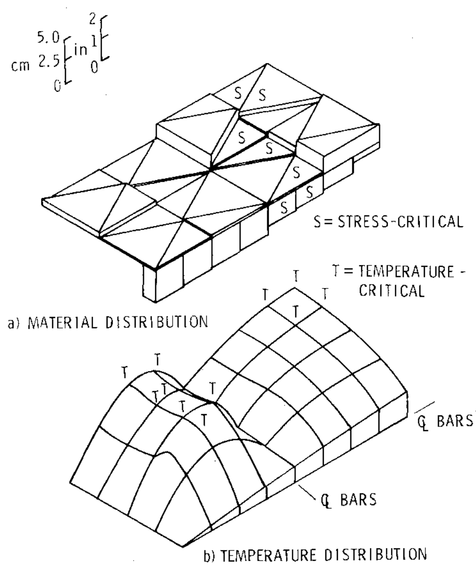


Fig. 5 Material and temperature distributions in optimum design of titanium plate with aluminum bars.

includes 24 membrane elements, 12 bar elements, and 61 grid points resulting in 36 design variables. Because of symmetry, only half of the structure is modeled.

For the case where $N_x = 210$ kN/m, the optimality criterion method with FSD yielded an optimum mass of 4.73 kg (10.42 lbm) and the mathematical programming procedure yielded an optimum mass of 4.72 kg (10.39 lbm). The designs obtained by the two methods are nearly identical and are governed primarily by the temperature constraints. None of the elements is strength-critical, but about one-third of the grid points on the plate are temperature critical (those farthest from the aluminum bars and the heat sink). No points on the bars are temperature critical.

For the case where $N_x = 6.30$ MN/m, the optimality criterion with FSD yielded an optimum mass of 4.80 kg (10.58 lbm) while the mathematical programming approach yielded an optimum mass of 4.79 kg (10.56 lbm). The design for this case is governed by both temperature and strength requirements. The material distribution in the final design is shown in Fig. 5a. The temperature distribution in the final design is shown in Fig. 5b. The elements which are stress critical are indicated by the symbol S in Fig. 5a and the grid

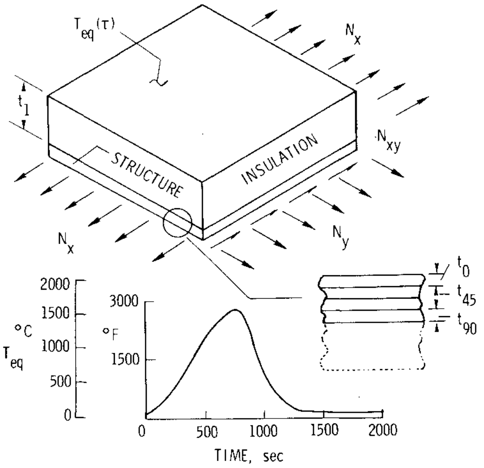


Fig. 6 Mathematical model and loadings for insulated composite panel.

points which are temperature critical are indicated by the symbol T in Fig. 5b. Again the designs obtained by the two different methods are essentially identical.

The designs by the optimality criterion method with FSD were obtained in 16 iterations (convergence to within 5% of final mass) and required only 22 s of execution time on CDC CYBER 175 computer. The mathematical programming procedure required about an order of magnitude more computing effort. The preceding results indicate that such alternate approaches can serve as useful tools along with the more general and rigorous mathematical programming procedures available for thermal structural optimization work.

Optimization of an Insulated Composite Panel with Transient Heating

All of the previous work has been concerned with steady-state loading and response. In the more difficult problem of design for transient loading, it is necessary to size a structure subject to constraints that must be satisfied over a period of time. Moreover, the analysis of each trial design entails the calculation of time histories of the appropriate response quantities such as stresses and temperatures. Clearly, optimization of complex structures for transient loading poses a challenge to existing structural optimization techniques.

Problem Definition and Approach

To illustrate some of the considerations involved in transient optimum design for a configuration of some practical significance, attention is directed to the problem of determining the minimum-mass design of the insulated panel shown in Fig. 6. A transient temperature T_{eq}, typical of a reentry heating trajectory is applied to the outer surface of the insulation layer. One-dimensional heat transfer is assumed, and the back of the panel is adiabatic. The structure is either metal or a balanced symmetric composite laminate with 0-, ±45- and 90- deg plies subjected to a general set of in-plane forces. The quantity to be minimized is the total mass per unit surface area, given by

$$w = \rho_I t_I + 2\rho_2 (t_0 + t_{45} + t_{90}) \tag{25}$$

where t_I is the insulation thickness and t_0, t_{45}, and t_{90} are ply thicknesses. Constraints are imposed on the structure to prevent excessive temperatures and structural failure over a period of time $\bar{\tau}$. These constraints are given by

$$T(\tau) \leq T_a \quad 0 \leq \tau < \bar{\tau} \tag{26}$$

and

$$F=F_1\sigma_1+F_2\sigma_2+F_{11}\sigma_1^2+F_{22}\sigma_2^2+F_{66}\sigma_{12}^2\leq 1 \tag{27}$$

The constraint given by Eq. (27) corresponds to the tensor polynomial or Tsai-Wu theory[24] and must be satisfied for all pertinent values of time in each of the four layers. The constraints may be cast in nondimensional form as follows

$$g_1(\tau)=T(\tau)/Ta-1\leq 0 \qquad 0\leq\tau\leq\bar{\tau} \tag{28}$$

$$g_{j+1}(\tau)=F_j(\tau) \qquad j=1,2,\ldots,N\text{ layer} \tag{29}$$

The accommodation of time-varying constraint equations presents analytical difficulties. An investigation of the optimization of an ablating heat shield[11] suggests using time-integrated averages of the constraint equation whereby Eq. (28) is replaced by

$$g=(T^*/T_a)-1\leq 0 \tag{30}$$

where

$$T^*=\frac{1}{\bar{\tau}}\int_0^{\bar{\tau}}T(\tau)\mathrm{d}\tau \tag{31}$$

It was noted in Ref. 11, however, that use of the integrated averages tends to smooth the penalty term and causes the temperature to be somewhat insensitive to changes in the design variables. An alternative approach being tried in the present work is to satisfy the constraints at a number of specified times. Thus, the constraints of Eqs. (28) and (29) are replaced by the following:

$$g_i=T(\tau_i)/T_a-1\leq 0 \qquad i=1,\ N\text{times} \tag{32}$$

$$g_{N\text{times}+ij}=F_j(\tau_i)-1\leq 0 \qquad i=1,\ N\text{times};\quad j=1,\ N\text{layer} \tag{33}$$

Thus, the total number of constraints is $(1+N\text{ layer})\times N$ times.

The optimization problem is solved by the AESOP program.[21] The transient temperature history is obtained by an analytical solution given in Ref. 25.

Application to Insulated Panels

The foregoing procedure was used to assess the relative merits of three different structural materials: graphite/polyimide (G/PI), graphite/epoxy (G/E), and aluminum. Constant thermal properties given in Table 4 were used. Mechanical properties were assumed to vary linearly between their values at room temperature and at the respective allowable temperature (see Table 2). The properties in Table 2 are taken from Refs. 26, 27, and 28, respectively.

The loads on the structure are as follows: $N_x=-700$ kN/m (−4000 lbf/in.), $N_y=228$ kN/m (1300 lbf/in.), and $N_{xy}=22.4$ kN/m (128 lbf/in.). The allowable temperatures are 177°C (350°F) for G/E and aluminum and 260°C (500°F) for G/PI. The heating duration is 2000 s and the monitoring time ($\bar{\tau}$) is 6000 s.

Before proceeding with the applications, a study was performed to establish the effect on the design of the number of discrete times at which the constraints were imposed. The results of this study for the G/E structure indicate that for this example only a few times (3 or 4) are necessary for an adequate design. It is recognized however, that the number of discrete times required is somewhat problem dependent; and while 3-4 times is adequate for heat-sink configurations such as the insulated panel, more time steps may be required if the discrete-times approach is extended to more general configurations with more complex heating histories.

As summarized in Table 5, optimum designs were obtained for the three materials with and without mechanical loads. The results indicate that use of G/PI with its higher allowable temperature gives a lighter overall design than G/E or aluminum. Also G/E gives a lighter overall design than aluminum, because the savings in structural weight of G/E offsets the higher insulation weight required. For all three materials the insulation mass is less for the loaded case than for the unloaded case because the structural mass needed to carry the mechanical loads increase the heat-sink effectiveness of the structure and decreases the need for insulation.

For the designs given in Table 5, the structure is temperature critical for at least one point in time. For example, in the G/E panel the temperature was 177°C (350°F) at $\tau=3300$ s. When the G/E panel was redesigned for higher loads of $N_x=N_y$ 2.62 MN/m (15,000 lbf/in.) and $N_{xy}=1.75$ MN/m (10,000 lbf/in.) the structure was not temperature critical; the maximum temperature was 162°C (324°F). Therefore,

Table 4 Thermal properties used for optimization of insulated panels

Material	K W/M°C	K Btu/in. s °F	c J/kg°C	c Btu/lbm°F	ρ kg/m^3	ρ lbm/in.^{3}
Insulation	0.181	2.42×10^{-6}	1200	0.291	144	0.005208
G/E	1.64	2.2×10^{-5}	962	0.23	1550	0.056
G/PI	2.30	3.08×10^{-5}	1297	0.31	1550	0.056
Aluminum	234	$3{:}13\times10^{-3}$	920	0.22	2770	0.100

Table 5 Optimum mass distribution in insulated panels for various structural materials

Material	No load[a] Insulation mass	No load[a] Structure mass	No load[a] Total mass	Load[a] Insulation mass	Load[a] Structure mass	Load[a] Total mass
Graphite/epoxy	16.6	3.08	19.7	16.5	3.17	19.7
$Ta=177$°C (350°F)	(3.40)	(0.63)	(4.03)	(3.39)	(0.65)	(4.04)
Aluminum	14.8	6.05	20.8	14.6	6.79	21.4
$Ta=177$°C (350°F)	(3.03)	(1.24)	(4.27)	(2.99)	(1.39)	(4.38)
Graphite/polyimide	11.7	3.56	15.2	11.4	3.86	15.3
$Ta=260$°C (500°F)	(2.39)	(0.73)	(3.12)	(2.34)	(0.79)	(3.13)

[a] All mass measures in kg/m^2 (lbm/ft^2).

contrary to intuition, the optimum design is not necessarily one in which the structure operates at its maximum service temperature. This observation was made previously for an ablation panel in Ref. 11 and has an important implication; namely, designing optimum structures for thermal applications requires simultaneous consideration of temperature and stress constraints with no a priori assumption that the optimum structure operates at its maximum use temperature.

Concluding Remarks

This paper presents techniques for optimum structural design for structures where temperature is an important design consideration. An improved variant of fully stressed design (FSD) denoted thermal fully stressed design (TFSD) is described which converges significantly faster than FSD for problems where thermal stresses are comparable in magnitude to mechanical stresses. Also, an optimality criterion is described for sizing structures subjected to temperature constraints.

Simultaneous requirements on strength and temperatures are approached by two different techniques: 1) state-of-the-art mathematical programming techniques with a general purpose optimizer, and 2) an alternate approach using the optimality criterion technique for temperature constraints combined with FSD for strength constraints. Both techniques are demonstrated for a titanium plate with aluminum bars subjected to mechanical loading and heating. Designs obtained by the two methods are in close agreement. The alternate approach requires an order of magnitude less computing effort to converge.

An investigation of transient problems focuses on the optimization of the insulation and ply thicknesses for an insulated composite panel with a time-varying temperature applied to the outer surface of the insulation and a general set of in-plane loads applied to the panel. Temperature and strength constriants in this problem are satisfied at a finite number of time points. Some results from the foregoing procedure are presented for insulated graphite/epoxy, graphite/polyimide, and aluminum panels.

Extension of analytical design techniques to complex structures especially with transient heating and loading continues to pose a challenge to structural optimization technology. This paper summarizes initial attempts to focus the technology on problems which embody the essence of designing for extreme temperature applications. Among the areas of work which require additional development are design-oriented thermal reanalysis techniques, improved treatment of constraints on time-varying response variables, and automatic identification of times in a flight trajectory when critical loads occur.

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